

Study Guide

Exponential Functions

03/01/2012

Exponential Functions

An exponential function is an equation that has a variable in the exponent. The functions below are all exponential functions, where $f(x)$ represents the letter y and is dependent on the value of x .

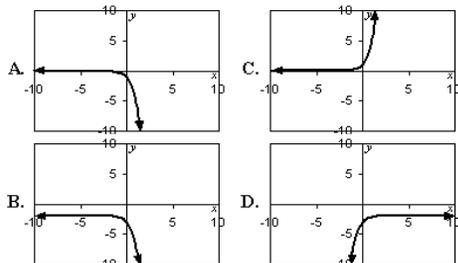
$$f(x) = 2^x$$

$$f(x) = 5^{2x+1}$$

$$f(x) = 4 - 6^{3x}$$

When x -values are substituted into the equation and the right side of the equal sign is simplified, a list of $f(x)$ (or y) values is obtained. This process gives you a set of (x, y) coordinate pairs that can be plotted (or graphed) on a coordinate plane.

Example 1: Which graph represents the exponential function $y = -2 - 5^{-x}$?



Step 1: Make a table of values with the following column headings.

x	$y = -2 - 5^{-x}$	(x, y)

Step 2: Choose values to enter into the x column (remember to choose the smallest possible values to keep computations as simple as possible). It is generally necessary for the values to contain both negative and positive numbers and the number zero. As few as three values could be used, but this example will use these five values: $-2, -1, 0, 1, 2$. Enter the values into the table.

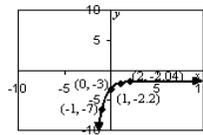
Step 3: Substitute each x -value into the equation in the center column for x and simplify. This will determine the y -values for the coordinate points.

$x = -2$	$x = -1$	$x = 0$
$y = -2 - 5^{-(-2)}$	$y = -2 - 5^{-(-1)}$	$y = -2 - 5^{-(0)}$
$y = -2 - 5^2$	$y = -2 - 5^1$	$y = -2 - 5^0$
$y = -2 - 25$	$y = -2 - 5$	$y = -2 - 1$
$y = -27$	$y = -7$	$y = -3$
$x = 1$	$x = 2$	
$y = -2 - 5^{-(1)}$	$y = -2 - 5^{-(2)}$	
$y = -2 - 5^{-1}$	$y = -2 - 5^{-2}$	
$y = -2 - \frac{1}{5}$	$y = -2 - \frac{1}{25}$	
$y = -2\frac{1}{5} = -2.2$	$y = -2\frac{1}{25} = -2.04$	

Step 4: Write the (x, y) pairs generated in Step 3 into the (x, y) column, using the values from the x column and the y -values from the center column.

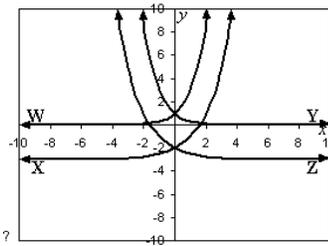
x	$y = -2 - 5^x$	(x, y)
-2	-27	$(-2, -27)$
-1	-7	$(-1, -7)$
0	-3	$(0, -3)$
1	-2.2	$(1, -2.2)$
2	-2.04	$(2, -2.04)$

Step 5: Plot the points on a coordinate plane to graph the exponential function.



Step 6: Compare this graph to the solution choices and choose the correct answer, which is choice D.

Answer: D.



Example 2: Which graph represents the exponential function $y = 3^x$?

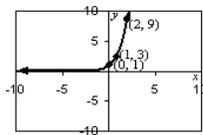
Step 1: Make a table of values and choose x -values to substitute into the equation.

x	$y = 3^x$	(x, y)
-2		
-1		
0		
1		
2		

Step 2: Substitute the values in the x column into the equation in the center column to determine the corresponding y -values. Write the coordinate points in the (x, y) column.

x	$y = 3^x$	(x, y)
-2	$1/9 = 0.\bar{1}$	$(-2, 0.1)$
-1	$1/3 = 0.\bar{3}$	$(-1, 0.3)$
0	1	$(0, 1)$
1	3	$(1, 3)$
2	9	$(2, 9)$

Step 3: Plot the points on a coordinate plane to graph the exponential function.



Step 4: Compare this graph to the curves in the question and choose the correct answer.

Answer: W

Comparing Graphs of Exponential Functions:

The standard form for an exponential function is shown below.

$$y = ab^x$$

For purposes of comparing graphs of exponential functions, two more variables need to be added, such that the standard form becomes the form shown below.

$$y = ab^{(x+d)} + c$$

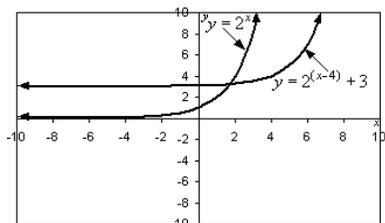
The value of c determines whether the graph shifts upward or downward and the value of d determines whether the graph shifts right or left. See the table below.

Variable	Result to Graph
$c > 0$	graph shifts upward $ c $ units
$c < 0$	graph shifts downward $ c $ units
$d > 0$	graph shifts to the left $ d $ units
$d < 0$	graph shifts to the right $ d $ units

Example 3: Compare the graph of $y = 2^{(x-4)} + 3$ to the graph of $y = 2^x$.

Step 1: In $y = 2^{(x-4)} + 3$, $c = 3$ and $d = -4$. **Step 2:** Look at the table above to determine that the graph of the exponential function should move *upward* 3 units and to the *right* 4 units.

Step 3: Compare the actual graphs below to see if these predictions were right.



An activity that will help reinforce the skill of graphing exponential functions is to explore exponential functions with the student using a graphing calculator. Make up a list of exponential functions like the one below.

$$y = 5^x$$

$$y = 5^{-x}$$

$$y = 5^x + 6$$

$$y = 5^x - 6$$

Follow the steps to graph each function using a graphing calculator. Notice the way changing a sign or adding a term alters the graph.

Step 1: Follow the instructions in your graphing calculator manual to set the viewing window to the following values: $x \text{ min} = -10$, $x \text{ max} = 10$, $x\text{scl} = 1$; $y \text{ min} = -10$, $y \text{ max} = 10$, $y \text{ scl} = 1$. Exit the window.

Step 2: Press the $y =$ key to open the equation editor. Delete any equations that are listed. Enter the following keystrokes for the first exponential equation on the list: 5^x . Press **Enter**.

Step 3: Press the **Graph** key to display the graph of the function.

****NOTE:** This activity could be completed using graph paper and tables of values to graph the exponential functions.